

Periodic Motion Experiment

by

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1. Introduction

The motion of Earth can be described as rotation about an axis while undergoing circular motion about the Sun. Earth's motion is repetitive. It takes approximately 24 hours for Earth to rotate through 360° about its axis. One complete revolution about the sun takes approximately 365 and one third days, i.e., one solar year. Because the motion repeats itself, we refer to it as *periodic motion*. The periodic nature of Earth's motion is vital to our continued existence. The naturally occurring motion keeps Earth at a distance of approximately 93 million miles from the Sun. If Earth were to move into a smaller orbit, such as that of Venus, the heat would be enough to wipe out most, if not all, life forms on the planet. If Earth were to move into a larger orbit, such as that of Mars, we would not receive sufficient heat energy from the Sun to sustain life. Earth's motion is just one particular example of periodic motion, many other types exist. In this experiment we will formally define periodic motion and describe a few examples.

2. Background

The requirement that the motion of an object must satisfy to be periodic is that it possess a definitive *period*. The period is the time that it takes for the object to travel through one complete cycle. In our example of Earth's motion about the Sun, the period, T , of Earth's circular motion is one solar year. We define the *frequency*, f , as the number of cycles an object travels per unit time, as

$$f = \frac{1}{T}. \quad (1)$$

Additionally, we define the *angular frequency*, ω , as

$$\omega = 2\pi f. \quad (2)$$

When T is measured in seconds, units of f are sec^{-1} , or equivalently, Hz. In addition, units of ω are rad/sec.

Before considering other properties of periodic motion, we will define the types of energies in a conservative system. We have previously studied the energy of an object on a microscopic level in our discussion on heat. In this discussion we will concentrate on energy on the macroscopic level. We broadly categorize the types of energy in a conservative system as being either *potential* or *kinetic* energy, where the total energy of the system, E , is the sum of the potential and kinetic energy. Potential energy is a relative quantity that depends on the spatial arrangement of the object in a system of bodies that exert forces on each other expressed with a coordinate frame. Kinetic energy is associated with the motion of the object. Our system consists of an object in motion and a coordinate frame within which we describe the motion. If the total energy of the system remains constant, such that any loss in the object's potential energy is completely made up by an increase in the object's kinetic energy and vice versa, then the object's motion can be periodic. See [4] for details. We will now focus on specific cases of periodic motion.

Spring–mass system

The force, F_s , exerted by a spring on a mass that has been either compressed or stretched, by a small displacement, is given by Hooke's Law

$$F_s = -kx \quad (3)$$

where k is a positive constant known as the force constant or stiffness of the spring, and x is the displacement of the spring from its *natural* length. The natural length of a spring is the length of the spring when no external forces are applied to it. The system is at equilibrium when the sum of all forces acting on the system is zero. The potential energy, U_s , stored in a spring obeying Hooke's Law is obtained from

$$\begin{aligned} dU_s &\equiv \int_0^x (kq) dq \\ \Rightarrow U_s &= \frac{1}{2} kx^2 \end{aligned}$$

where q is a dummy variable. The kinetic energy, K , of any object is given as

$$K = \frac{1}{2} mv^2$$

where m is the mass of the object and v is the velocity at which it is traveling. Consider the system shown in Figure 1. A one-dimensional coordinate frame is fixed to the location at which the spring is at its natural length. Hence, the equilibrium position of the mass coincides with the origin of the coordinate frame. The total energy of this system is the sum of the kinetic energy of the mass and the potential energy in the spring.

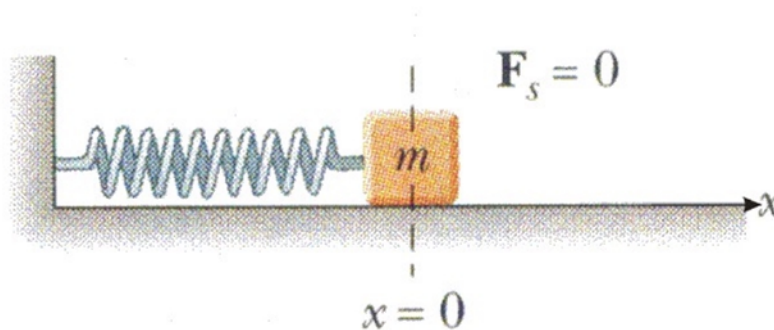


Figure 1: Spring–mass system

If we displace the mass to the right by a small distance, A , the spring will exert a force on the mass. If we hold the mass in place, implying $v = 0$, the total energy of the system will be the potential energy stored in the spring. This scenario is shown in Figure 2. The force acts in the opposite direction as that of the displacement as indicated in the figure. The negative sign given in (3) indicates that the spring force is a *restoring* force, one that acts to restore the spring to its natural length. The spring in Figure 2 has been

stretched; therefore the force will act to relax the spring. If we had instead compressed the spring, the force would act to expand the spring.

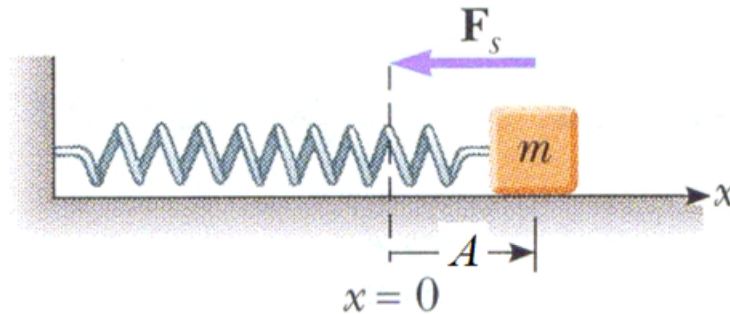


Figure 2: Spring–mass system

If we simply release the spring, the mass will have no initial velocity. However, the potential energy that was stored in the spring will be converted into kinetic energy. When the mass returns to the equilibrium position, the potential energy of the spring will be zero since the displacement of the spring is zero. At this instant the total energy of the system is the kinetic energy of the mass, which will be moving with its greatest speed. The mass will not stop instantaneously however. Rather, it will continue traveling to the left, towards the wall. The motion of the mass will then be impeded by the spring force. Therefore, the mass will decelerate until it has been displaced a distance equal to that of the initial displacement A . At that time the kinetic energy of the mass will have been fully converted back into potential energy in the spring. The mass will continue to oscillate in this manner. The maximum displacement of the spring is defined as the *amplitude* and is equal to the initial displacement, A , of the spring.

This conservative system as we have described it does not experience any loss in total energy. In a real system, such a description is overly simplified. The total energy of the system will decrease with time due to friction at the contact of the mass and the surface. Additionally, the stretching and relaxing of the spring will result in further losses to the total energy of the system.

A more complicated example of a spring–mass system is one that oscillates in a vertical frame of reference. Gravity cannot be ignored in this type of system; rather, it is an integral part of the system's equilibrium position. The equilibrium position for this type of system will, in general, not coincide with the natural length of the spring. Instead, the spring will be elongated such that the spring force is equal in magnitude to the gravitational force acting on the mass. See [4] for details on spring–mass systems.

Mass on a spring (pendulum)

Another common example of periodic motion is the pendulum. Consider the system shown in Figure 3. A mass is attached to a mass-less bar of constant length l . As the bar rotates, the mass will move along an arc that is part of a circle of radius l . We refer to the displacement along the arc, relative to the vertical reference, as the *arc length*, s . However, it is easier to describe the motion of the mass through the angle, θ , that exists between the bar and the vertical reference, such that when $\theta = 0$ the mass is located on the vertical reference. The arc length and angle are related as follows

$$s = l\theta. \quad (4)$$

To help us analyze the forces acting in this system, we will fix a coordinate frame on the mass. This coordinate system is defined by the tangential and radial components of the object's motion.

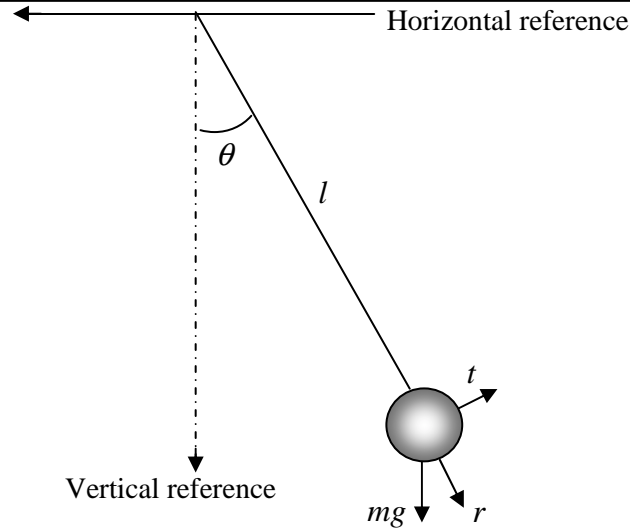


Figure 3: Pendulum

The motion of the pendulum is restricted to the tangential direction since the length of the bar is constant. Therefore, we will only concern ourselves with the forces that act in the tangential direction. The sum of all forces in the tangential direction, F_t , is

$$ma_t = \sum F_t = -mg \sin \theta \tag{5}$$

where a_t is the acceleration of the mass along the tangential direction given by $a_t = \frac{d^2s}{dt^2}$ and g is the free-fall acceleration due to gravity. Since l is a constant, substitution of (4) into (5) and dividing through by ml yields

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l} \sin \theta. \tag{6}$$

If we restrict our discussion to small angles then we can employ the *small angle approximation* given by

$$\sin \theta \approx \theta. \tag{7}$$

Substituting (7) into (6) leads to

$$\frac{d^2\theta}{dt^2} = -\omega^2\theta. \tag{9}$$

where ω is the angular frequency defined as follows

$$\omega \equiv \sqrt{\frac{g}{l}}. \tag{10}$$

We substitute (10) into (2) to obtain

$$f = 2\pi\sqrt{\frac{g}{l}}. \quad (11)$$

Furthermore, we substitute (11) into (1) to obtain

$$T = 2\pi\sqrt{\frac{l}{g}}. \quad (12)$$

As in the example of the spring–mass system we ignore the effects of friction. See [4] for details on pendulums.

3. Equipment List

Board of Education (BOE) with Basic Stamp 2 (BS2)

The combination of the BS2 embedded within the BOE will serve as the microcontroller that monitors the experiments that you are about to do. The BS2 is a 24 pin Dual Inline Package (DIP) integrated circuit (IC). It is based on Microchip Inc.'s PIC 16C57 microcontroller. The BS2 is powered by a 6-14V direct current (VDC) power supply. An onboard voltage regulator provides a steady 5VDC output to the BS2. The BS2 comes with ROM, 2KB Electronically Erasable Programmable ROM (EEPROM), and a small amount of RAM. The BS2 is programmed in PBasic language, the instruction set that is stored in the BS2 ROM. The user defined program is downloaded into the EEPROM from a PC to the BOE using a DB-9 serial cable. The excess EEPROM can be used for long term data storage. The BS2 has 16 general purpose digital input/output (I/O) pins that are user defined. The high position on a digital I/O pin refers to a 5VDC and a low position on a digital I/O pin refers to a 0VDC (ground potential). Each pin can source (supply) a maximum current of 20mA and sink (draw) a maximum current of 25mA. The 16 I/O pins on the BS2 at any given time can source/sink a maximum of 40mA/50mA. If using an external 5VDC voltage regulator, these limits apply to each group of 8 pins, P0-P7 and P8-P15. Exceeding these current source/sink limits or establishing a voltage on a pin greater than 5VDC will damage the BS2. See [2, 3] for details.

A limitation that often arises when using the BS2 is the lack of support for floating point variables. Utilizing floating point operations like division in the absence of floating point variables may lead to mathematical errors due to truncation. The largest variable or constant that can be stored on the BS2 is of *word* size (16 bits), which has a numerical range of 0-65,535 in decimal notation.

A BOE and BS2 are depicted in Figure 4 [1]. The BS2 is placed, in the same orientation as shown, in the IC socket in the lower left corner of the BOE.

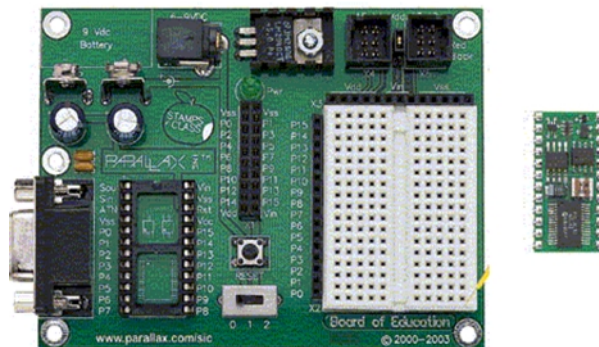


Figure 5: Parallax Board of Education with a Basic Stamp 2 at right

Pendulum experiment test bed

The pendulum experiment test bed will be used to measure the periodicity of a simple pendulum. It consists of a fixed mass attached to a length of rope, an IR transmitter and receiver pair, and a servo motor. The length of the pendulum is adjusted by the servo motor through a pulley. The length of the pendulum in this experiment can be set to one of two lengths, either 18.5cm or 25cm. The IR transmitter and receiver have been placed on the vertical reference such that when the mass lies on the vertical reference the signal is interrupted. The program that was written for the pendulum experiment first prompts the user to set the length of the pendulum. Then the user is prompted to set the mass in motion. The program then counts the number of times the IR signal has been interrupted within a fixed period of time. For each period of motion the mass will interrupt the IR signal twice. The BS2 is extremely useful for measuring short intervals of time, on the order of milliseconds (ms), as it can perform thousands of operations per second. The actual time that elapses each time the program runs has been calibrated against a stopwatch. This time is used by the BS2 when it calculates the periodicity of the pendulum. The test bed connects to the BS2 through a DB-9 adapter. Figure 6 is a view of the pendulum experiment test bed. On the top of the figure the IR transmitter can be seen. The IR receiver can be seen at the bottom of the figure in line with the transmitter. In this figure the mass has been displaced to the right. As the mass swings through the vertical reference the signal from transmitter to receiver will be broken. The pulley used to adjust the length of the pendulum can be seen below and to the right of the IR transmitter. The DB-9 adapter that connects the test bed to the BS2 can be seen in the background of the figure.

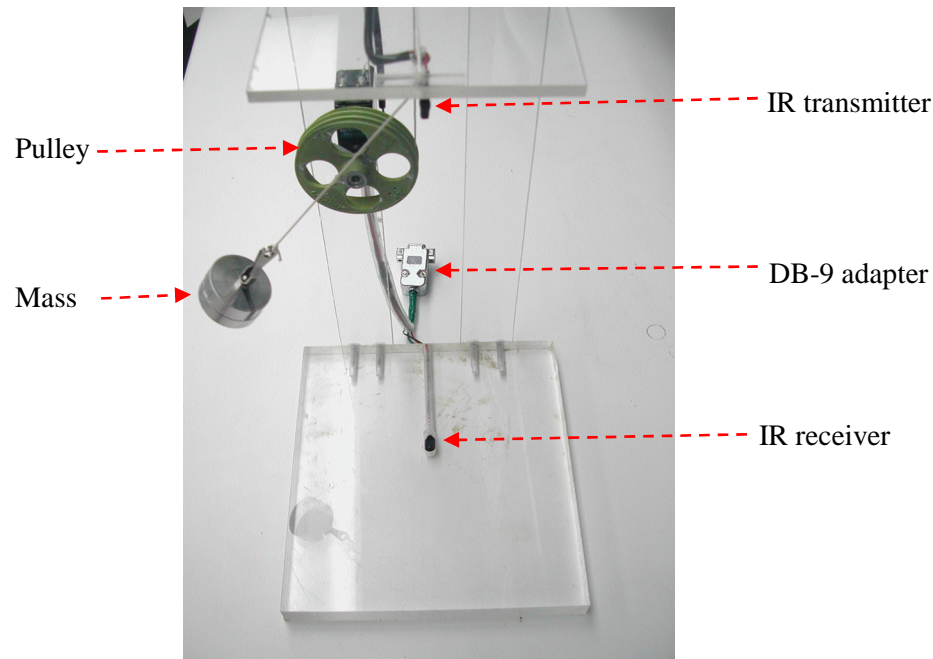


Figure 6: A view of the pendulum experiment test bed

Figure 7 is another view of the pendulum experiment. In this view the servo motor that is used to turn the pulley can be seen just behind the pulley in the upper left hand corner of the figure.

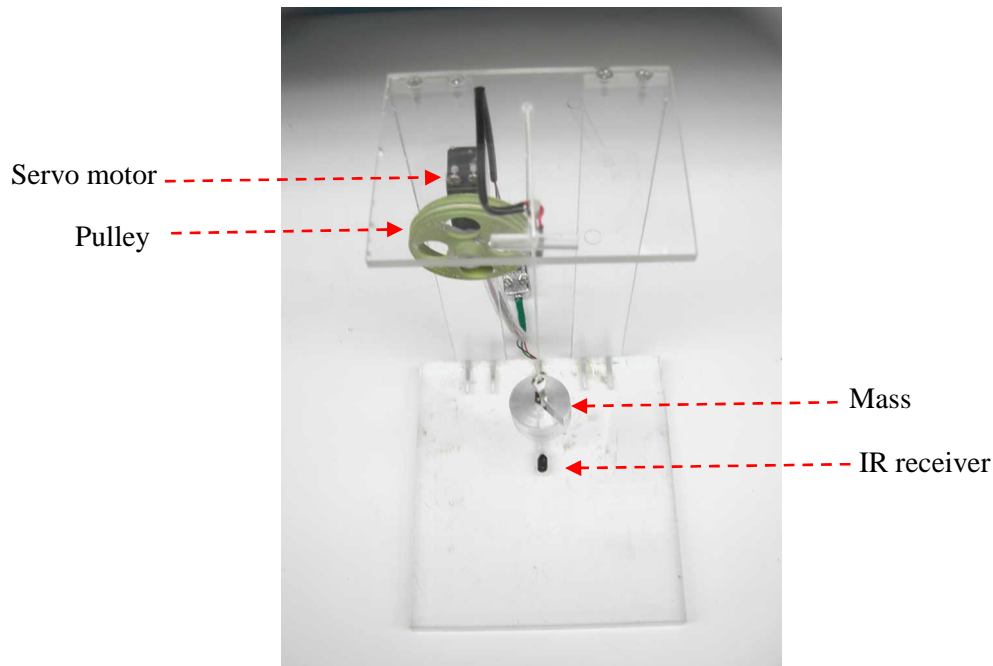


Figure 7: A view of the pendulum experiment test bed

4. Experimental procedure

4.1. Pendulum experiment

Goals:

1. Measure the period of oscillation for a pendulum.

Pendulum experiment procedure

1. Connect the BS2 to the test bed through the DB-9 adapter and to a computer through a DB-9 serial cable.
2. Download the *Pendulum.bs2* file to the BS2.
3. At the prompt enter set the length of the pendulum.
4. At the prompt set the mass in motion by displacing it in either direction.
5. The BS2 will track the number of times the mass passes through the vertical reference.
6. The BS2 will then calculate the periodicity of the pendulum.
7. Next, the BS2 will display the theoretical and experimental values for the period of the pendulum.
8. To continue with this experiment follow the prompts regarding restarting the experiment.

Discussion

Do the theoretical and experimental results agree? In what applications can this type of pendulum be used?

5. Suggested Project

- 1) Spring-mass system experiment: Construct an experiment that can measure the period of oscillation of a spring-mass system. Modify the design so that it is possible to measure the speed of the mass as it passes through the equilibrium position.

6. References

- [1] Online: Online: <http://www.parallax.com>, web site of Parallax, Inc. manufacturer and distributor of Basic Stamp microcontrollers containing technical specifications and user manuals.
- [2] Online: Online: <http://www.stampsinclass.com>, web site of Stamps in Class containing educational resources for Basic Stamp microcontrollers
- [3] V. Kapila, *Course Notes for Mechatronics ME 3484*, Polytechnic University, Brooklyn, NY, (2003).
- [4] R.A. Serway and R.J. Beichner, *Physics for Scientists and Engineers*. Saunders College Publishing, Orlando, FL, 5th Ed., (2000).

Appendix A: Circuit schematics

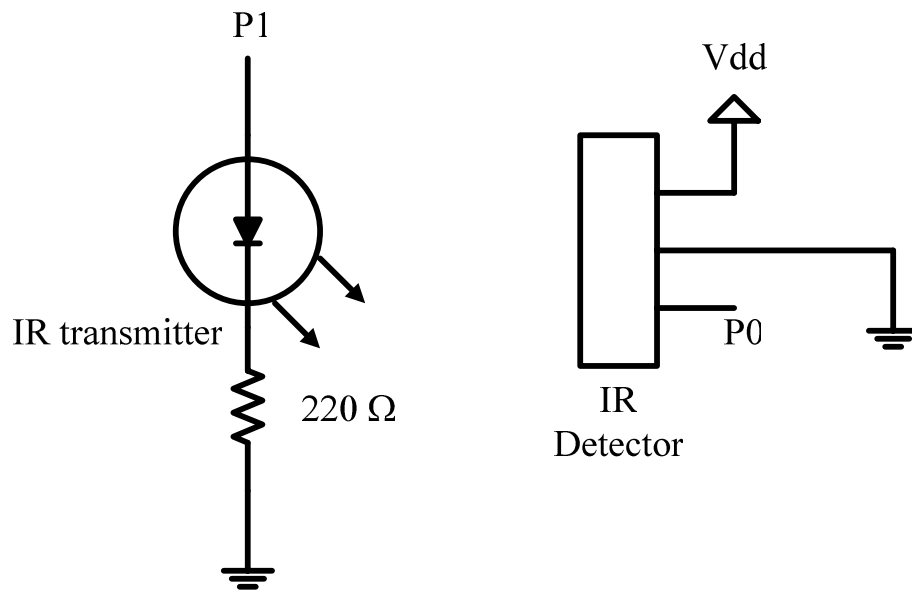


Figure A1: Circuit schematic of the IR transmitter/receiver pair used to detect motion

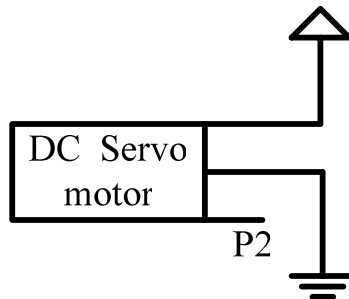


Figure A2: Circuit schematic for the servo motor used to adjust the length of the pendulum