Experiment 3: Modeling, Identification, and Control of a DC-Servomotor

Concepts emphasized: Dynamic modeling, time-domain analysis, system identification, and position-plus-velocity feedback control.

1. Introduction

DC-motors that are used in feedback controlled devices are called DC-servomotors [1–4]. Applications of DC-servomotors abound, e.g., in robotics, computer disk drives, printers, aircraft flight control systems, machine tools, flexible manufacturing systems, automatic steering control, etc. DC-motors are classified as *armature controlled* DC-motors and *field controlled* DC-motors [4].

This laboratory experiment will focus on the modeling, identification, and position control of an armature controlled DC-servomotor. In particular, we will first develop the governing differential equations and the Laplace domain transfer function model of an armature controlled DC-motor. Next, we will focus on the identification of the unknown system parameters that appear in the transfer function model of the DC-servomotor. Finally, we will develop and implement a position-plus-velocity, also known as proportional-plus-derivative (PD), feedback controller to ensure that the DC-motor angular position response tracks a step command.

2. Background

DC-motor modeling: A schematic representation of an armature controlled DC-motor is given in Figure 1. For an armature controlled DC-motor, the field current $i_{\rm f}$ is constant and the torque $T_{\rm m}$ generated at the DC-motor shaft is given by [2–4]

$$T_{\rm m} = K_{\rm T} i_{\rm a}, \tag{2.1}$$

where $K_{\rm T}$ is the given motor torque constant (N-m/Amp) and $i_{\rm a}$ is the armature current (Amp). Note that for an armature controlled DC-motor, the back e.m.f. induced in the armature due to armature rotation is directly proportional to the armature angular velocity $\omega_{\rm m}(t) \triangleq \frac{d\theta_{\rm m}}{dt}$ where $\theta_{\rm m}(t)$ is the angular position of the motor shaft. Thus, following [2–4]

$$V_{\rm b} = K_{\rm b}\frac{d\theta_{\rm m}}{dt}, \tag{2.2}$$

where $K_{\rm b}$ is a given motor constant (Volt-sec/rad).

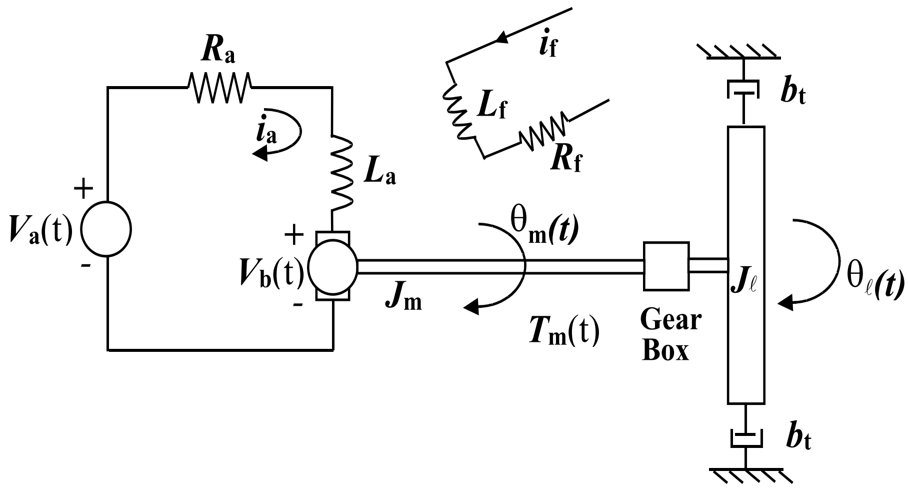

Figure 1: Armature Controlled DC-Motor

Next, note that the angular speed $\omega_{\rm m}(t)$ of an armature controlled DC-motor is controlled by the armature voltage $V_{\rm a}$. The differential equation relating the armature current $i_{\rm a}$ and the back e.m.f. $V_{\rm b}$ to the armature voltage $V_{\rm a}$ can be obtained by applying *Kirchhoff's Voltage Law* [1, 4]. In particular, according to the Kirchhoff's Voltage Law, at any given instant of time, the algebraic sum of voltages around any loop in any electric network is zero. Thus, a direct application of the Kirchhoff's Voltage Law to the armature circuit yields

$$L_{\rm a}\frac{di_{\rm a}}{dt} + R_{\rm a}i_{\rm a} + V_{\rm b} = V_{\rm a}. \tag{2.3}$$

Finally, we obtain the differential equation governing the motion of the mechanical load. First, note that in most applications, the DC-servomotor shaft is connected to a gear-box of a given gear-ratio $K_{\rm g}$ and the load is attached to the output shaft of the gear-box (e.g., see Figure 2). The gear-ratio $K_{\rm g}$ is give by $K_{\rm g} \triangleq \frac{n_\ell}{n_{\rm m}}$, where n_ℓ and $n_{\rm m}$ are the number of teeth on the load-side and the motor-side gears, respectively. It can be easily shown that the gear-ratio $K_{\rm g}$ relates the motor shaft angular position $\theta_{\rm m}$ to the gear-box output shaft angular position θ_ℓ by $K_{\rm g} = \frac{\theta_{\rm m}}{\theta_\ell}$. In addition, it can be shown that the load inertia J_ℓ acting at the output shaft of the gear-box when reflected at the motor shaft is given by $\frac{1}{K_{\rm g}^2}J_\ell$. Thus, an application of Newton's moment balance equation at the motor output shaft yields

$$J_{\rm m}\frac{d^2\theta_{\rm m}}{dt^2} + \frac{1}{K_{\rm g}^2}J_\ell\frac{d^2\theta_{\rm m}}{dt^2} + \frac{1}{K_{\rm g}^2}b_{\rm t}\frac{d\theta_{\rm m}}{dt} = T_{\rm m},$$

which can be rewritten as

$$J_{\mathrm{eq}}\frac{d^2\theta_\ell}{dt^2} + b_{\mathrm{t}}\frac{d\theta_\ell}{dt} \; = \; K_{\mathrm{g}}T_{\mathrm{m}}, \tag{2.4}$$

where $J_{\mathrm{eq}} = K_{\mathrm{g}}^2 J_{\mathrm{m}} + J_\ell$ is the total load inertia reflected at the motor shaft and b_{t} is the rotational viscous friction constant.

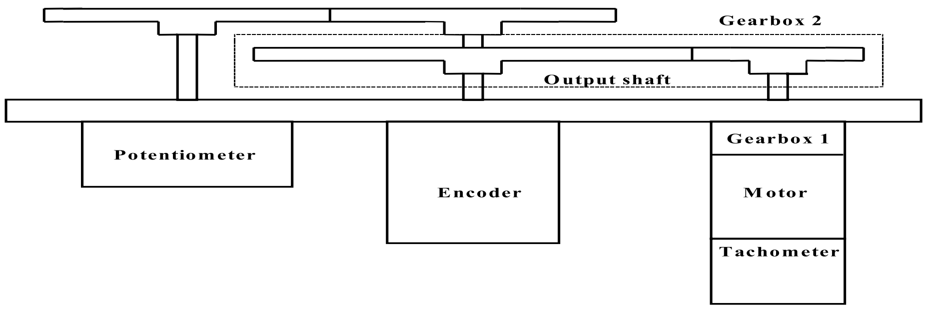

Figure 2: DC-Motor Experiment Test-Bed

Now, taking the Laplace transform of (2.1)–(2.4) and after some algebraic manipulations to eliminate the variables T_{m}, V_{b}, and i_{a} , we obtain

$$\frac{\theta_\ell(s)}{V_{\mathrm{a}}(s)} \; = \; \frac{K_{\mathrm{g}}K_{\mathrm{T}}}{s\left(L_{\mathrm{a}}J_{\mathrm{eq}}s^2 + (L_{\mathrm{a}}b_{\mathrm{t}} + R_{\mathrm{a}}J_{\mathrm{eq}})s + R_{\mathrm{a}}b_{\mathrm{t}} + K_{\mathrm{g}}^2K_{\mathrm{T}}K_{\mathrm{b}}\right)}. \tag{2.5}$$

In addition, the transfer function from input V_{a} to output ω_ℓ is given by

$$\frac{\omega_\ell(s)}{V_{\mathrm{a}}(s)} \; = \; \frac{K_{\mathrm{g}}K_{\mathrm{T}}}{L_{\mathrm{a}}J_{\mathrm{eq}}s^2 + (L_{\mathrm{a}}b_{\mathrm{t}} + R_{\mathrm{a}}J_{\mathrm{eq}})s + R_{\mathrm{a}}b_{\mathrm{t}} + K_{\mathrm{g}}^2K_{\mathrm{T}}K_{\mathrm{b}}}. \tag{2.6}$$

Now, assuming two real, simple roots of the characteristic equation of (2.6), *viz.*, p_{e} and p_{m}, partial fraction expansion of (2.6) yields

$$\frac{\omega_\ell(s)}{V_{\mathrm{a}}(s)} \; = \; \frac{K_{\mathrm{e}}}{s + p_{\mathrm{e}}} + \frac{K_{\mathrm{m}}}{s + p_{\mathrm{m}}}. \tag{2.7}$$

Next, using the inverse Laplace transform, the forced response of the system (with zero initial condition) to the input $V_{\mathrm{a}}(t)$ is given by

$$\omega_\ell(t) \; = \; \int_0^t \left[K_{\mathrm{e}}\mathrm{e}^{-p_{\mathrm{e}}(t-q)} + K_{\mathrm{m}}\mathrm{e}^{-p_{\mathrm{m}}(t-q)}\right] V_{\mathrm{a}}(q)dq. \tag{2.8}$$

In most practical applications of armature controlled DC-motors, $p_{\mathrm{e}} >> p_{\mathrm{m}}$; i.e., the electrical subsystem responds considerably faster than the mechanical subsystem. Hence, the first exponential term in (2.8) decays rapidly. Thus, the response $\omega_\ell(t)$ in (2.8) is dominated by the mechanical subsystem $\frac{K_\mathrm{m}}{s+p_\mathrm{m}}$. For simplicity, in DC-servomotor control applications the influence of the electrical subsystem component ($\frac{K_\mathrm{e}}{s+p_\mathrm{e}}$) on the response $\omega_\ell(t)$ in (2.8) is commonly neglected [2–4]. This can alternatively be viewed as neglecting the armature inductance effect, L_a. This simplification yields a first-order transfer function model which relates the DC-motor load angular velocity response ω_ℓ to the armature voltage input V_a, and is given by

$$\frac{\omega_\ell(s)}{V_\mathrm{a}(s)} = \frac{K_\mathrm{g} K_\mathrm{T}}{R_\mathrm{a} J_\mathrm{eq} s + R_\mathrm{a} b_\mathrm{t} + K_\mathrm{g}^2 K_\mathrm{T} K_\mathrm{b}}. \tag{2.9}$$

Before proceeding, note that, it can be shown that in the SI-Units used for K_T and K_b, the numerical values of K_T and K_b are identical [3]. Finally, the transfer function model of (2.9) can be equivalently written as

$$\frac{\omega_\ell(s)}{V_\mathrm{a}(s)} = \frac{K}{\tau s + 1}, \tag{2.10}$$

where K and τ are the dc-gain and the mechanical time-constant of the DC servomotor, respectively.

3. Objective

i) Analysis of DC-motor sensor characteristics.

ii) DC-motor system identification.

iii) PD control of the DC-motor to achieve the desired angular position step response characteristics.

4. Equipment List

i) PC with MultiQ-3 data acquisition card and connecting board

ii) Software environment: Windows, Matlab, Simulink, RTW, and WinCon

iii) SRV-02 DC-motor apparatus (See Figure 3) with potentiometer, optical encoder, and tachometer

iv) Universal power module: UPM-1503

v) Set of leads

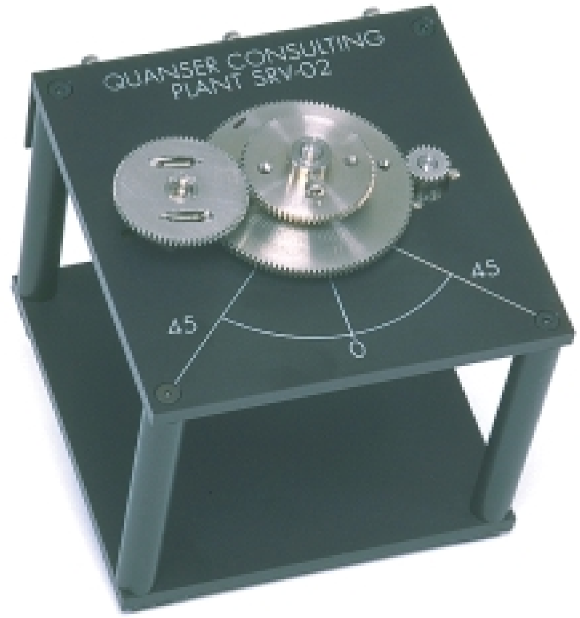

Figure 3: SRV-02 DC-Motor Apparatus

5. Experimental Procedure

i) Using the set of leads, universal power module, SRV-02 DC-motor apparatus, and the connecting board of the MultiQ-3 data acquisition card, complete the wiring diagram shown in Figure 4.

ii) Start Matlab and WinCon Server. In the Matlab window, at the command prompt, type "Experiment3" and hit the **Enter** key. This Matlab script will change the directory from the default Matlab directory to the directory where all files needed to perform Experiment 3 are stored.

iii) You can now perform various steps of the DC-motor identification and control experiment. However, before proceeding, you **must** request your laboratory teaching assistant to check your electrical connections.

iv) From the **File** menu of the WinCon Server, select the option **Open** to load the experiment file "Experiment3_Pot.wcp." This will load the files for determining the gain of

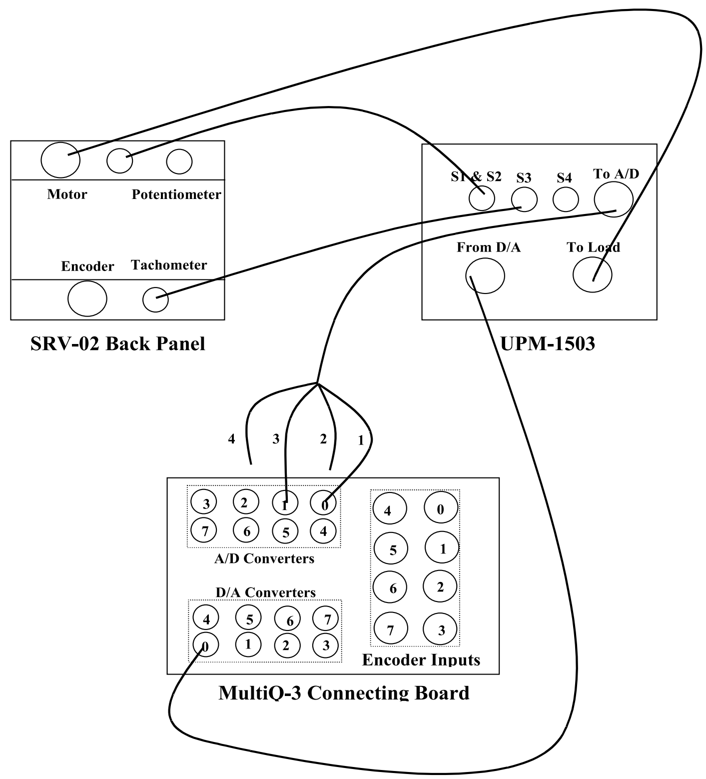

Figure 4: Wiring Diagram for DC-Motor ID and Control

the potentiometer K_{pot} $(\frac{\text{radian}}{\text{Volt}})$. A digital meter window will also appear on your desktop. The potentiometer gain K_{pot} relates the potentiometer output voltage V_{pot} to the load angular displacement θ_ℓ by $\theta_\ell = K_{\text{pot}} V_{\text{pot}}$. Next, from the **Window** menu of the WinCon Server, select the option **Simulink**. This will load the Simulink block-diagram "Experiment3_Pot.mdl" shown in Figure 5 to your desktop.

a) In the WinCon Server interface, click the green **Start** button to acquire the potentiometer voltage response.

b) Rotate the load connected to the output shaft (center gear) until the potentiometer voltage in the digital meter window shows 0 Volt. Please ensure that you get continuous variation in the neighborhood of this 0 Volt reading. If you note a discontinuity in the reading, turn the load by 180° and this will provide you close to 0 Volt reading. Read the angular position θ_0° of the load, corresponding to the 0 Volt potentiometer reading, off the protractor marked on the SRV-02 apparatus.

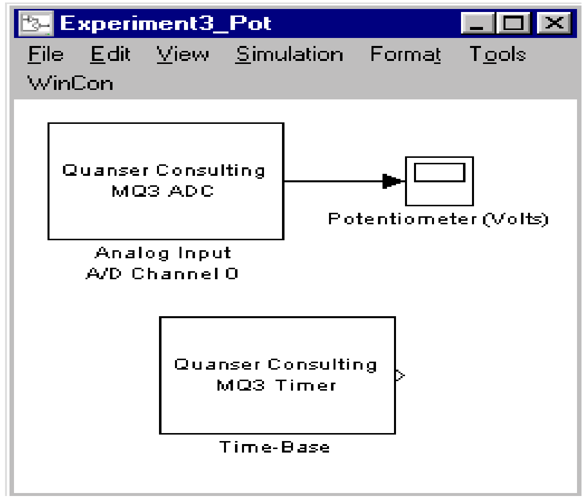

Figure 5: Simulink Block-Diagram for Determining Potentiometer Gain

c) Rotate the load to $\theta_0 + 90°$ and note the corresponding potentiometer voltage reading in the digital meter window.

d) Rotate the load to $\theta_0 - 90°$ and note the corresponding potentiometer voltage reading in the digital meter window.

e) In the WinCon Server interface, click the red **Stop** button when you finish collecting the potentiometer voltage response data.

v) Close the currently open digital meter window and the Simulink diagram. From the **File** menu of the WinCon Server, select the option **Open** to load the experiment file "Experiment3_Tach.wcp." This will load the files for determining the gain of the tachometer K_{tach} ($\frac{\text{radian}}{\text{second-Volt}}$) and a plot window. The tachometer gain K_{tach} relates the tachometer output voltage V_{tach} to the load angular velocity ω_ℓ by $\omega_\ell = K_{\text{tach}} V_{\text{tach}}$. Next, from the **Window** menu of the WinCon Server, select the option **Simulink** which loads the Simulink file "Experiment3_Tach.mdl" shown in Figure 6 to your desktop.

a) In the WinCon Server interface, click the green **Start** button. This applies a constant 1 Volt input to the DC-motor.

b) Measure the steady-state load angular speed and the corresponding steady-state tachometer output voltage reading in the plot window. **Hint:** Find the time re-

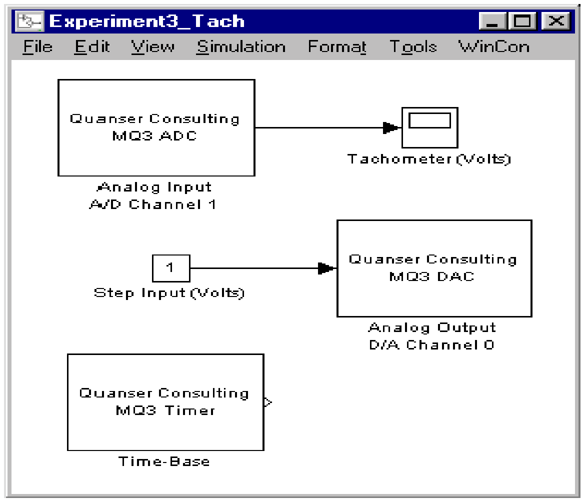

Figure 6: Simulink Block-Diagram for Determining Tachometer Gain

quired for 20 complete revolutions of the load and the corresponding steady-state tachometer output voltage reading at the end of 20 revolutions.

c) In the WinCon Server interface, click the red **Stop** button when you finish collecting the tachometer voltage response data.

vi) Close the currently open plot windows and the Simulink diagram. From the **File** menu of the WinCon Server, select the option **Open** to load the experiment file "Experiment3_DCID.wcp." Next, from the **Window** menu of the WinCon Server, select the option **Simulink** which loads the Simulink file "Experiment3_DCID.mdl" shown in Figure 7 to your desktop. In this diagram, the gain K_{tach} must be supplied by you. Run this part of the experiment to acquire the transient and steady-state angular velocity step response of the DC-motor under load.

vii) Close the currently open plot windows and the Simulink diagram. From the **File** menu of the WinCon Server, select the option **Open** to load the experiment file "Experiment3_PDCont.wcp." Next, from the **Window** menu of the WinCon Server, select the option **Simulink** which loads the Simulink file "Experiment3_PDCont.mdl" shown in Figure 8 to your desktop. In this diagram, the gains K_{pot} and K_{tach} must be supplied by you. In addition, the gains K_{P} and K_{D} must be designed and supplied by you. In particular, design a PD feedback controller so that the DC-motor angular position step response ex-

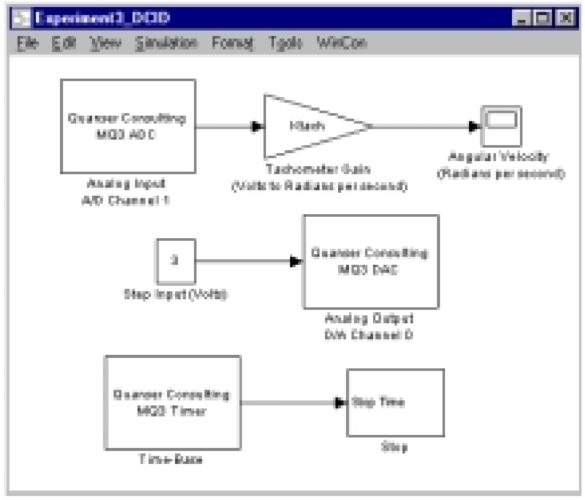

Figure 7: Simulink Block-Diagram for DC-Servomotor System Identification

hibits a peak overshoot $M_{\mathrm{p}} \leq 5\%$ with settling time $T_{\mathrm{s}} \leq 1$ second. The feedback diagram of the DC-motor with the PD feedback controller is shown in Figure 9. The characteristic equation of the closed-loop system in Figure 9 can be used for the purpose of finding K_{P} and K_{D} such that the desired performance specifications are achieved. Before proceeding, you **must** request your laboratory teaching assistant to approve your gain values. Run the experiment to record the angular position step response of the DC-motor.

6. Analysis

i) Calculate K_{pot} and K_{tach} from the experimental data collected in steps *iv*) and *v*) of Section 5.

ii) Analyze the open-loop angular velocity step response obtained in step *vi*) of Section 5 to determine the dc-gain K and the mechanical time constant τ of the DC-servomotor system.

iii) Obtain the angular velocity step response of the first-order system (2.10) with the parameters K and τ obtained in step *ii*) above. Compare the simulated angular velocity step response with the experimental response obtained in step *vi*) of Section 5 and comment.

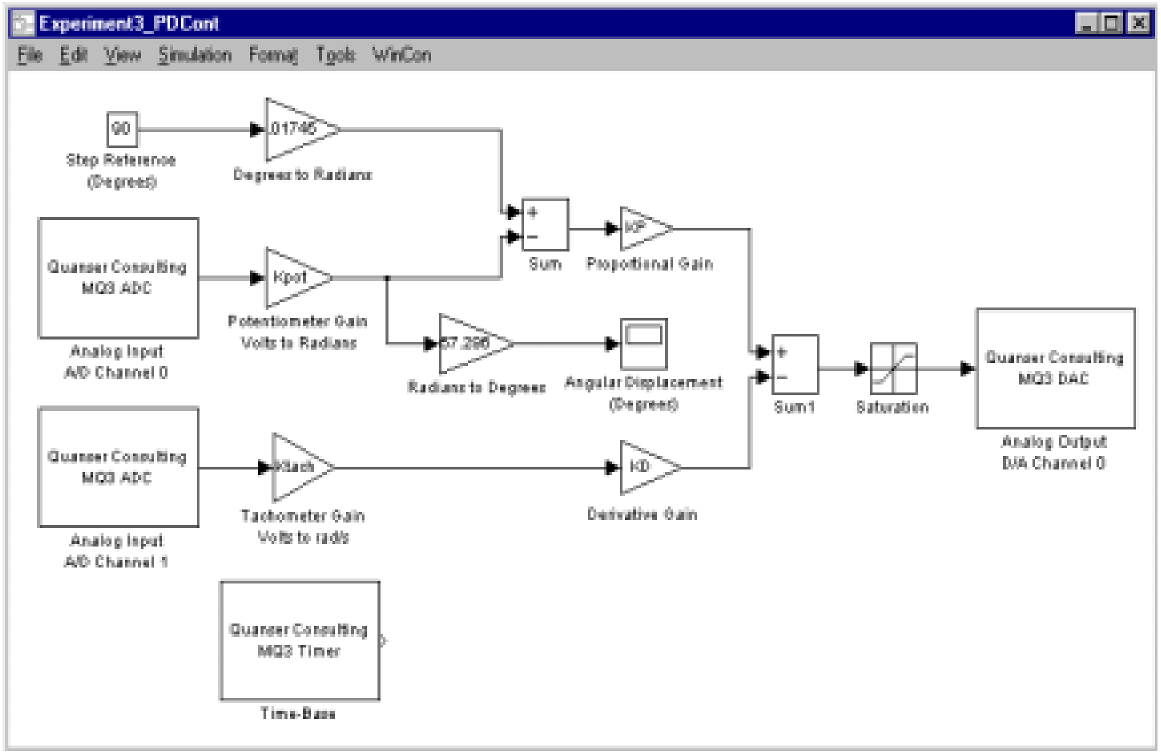

Figure 8: Simulink Block-Diagram for DC-Servomotor PD Control

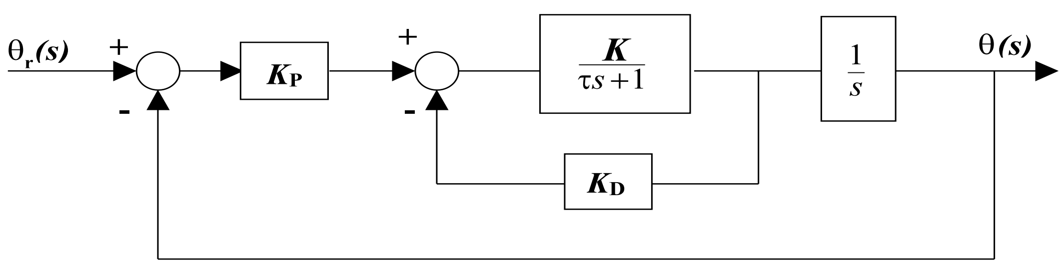

Figure 9: Closed-Loop Feedback Interconnection for PD Control of a DC-Motor

iv) Analyze the closed-loop angular position step response obtained in step *vii*) of Section 5 to determine if the performance specifications are satisfied. Comment on your results.

v) Design a proportional-integral-derivative (PID) controller so that the performance requirements specified in Section 5 are satisfied. Simulate the closed-loop angular position step response with the PID controller. Compare with the experimental closed-loop angular position step response obtained using the PD controller.

References

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